THE IMPACT OF USING ROBUST MM-ESTIMATOR IN REGRESSION MODELS: AN APPLICATION OF NIGERIAN INFLATION ON THE ECONOMIC GROWTH

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Abstract

This study was conducted to show the impact of using a Robust MM-Estimator in Econometric regression models, as an alternative to the Ordinary least squares(OLS) technique for analyzing the effects of inflation on the economic growth of Nigeria from 1980-2019, due to the inefficiency of the OLS in the presence of outliers. The study used annual time series data obtained from the Central Bank of Nigerian statistical bulletin. The study also reviewed the performance of OLS and compared it with that of the MM-estimator. The study used GDP as the dependent variable and also as a proxy for economic growth while Inflation rate, exchange rate and interest rate were used as the independent variables. The findings of the study showed that MM-estimation gives a better result than the OLS having the smallest RSE (0.106) and larger R-square (0.78) than the OLS with RSE (0.1205) and R-square (0.77). The result of the MM-estimation model showed that inflation has a negative (-0.07551) impact on the economic growth of Nigeria, while the control variables exchange rate and interest rate have a positive impact on the economy. The study also recommended that to tackle the problem of outlier contamination affecting econometric models, particularly on Inflation-Growth Nexus analysis, is to adopt the use of the MM-Estimation method as an alternative to the traditionally used OLS technique, because it can cope with the presence of outliers in the economic data vertically and horizontally. Likewise, it will also correct the violation of classical assumptions automatically.

Keywords: MM-estimation, OLS, Breakdown point, Efficiency, Inflation, Economic Growth.

Introduction

MM estimation is a special type of M-estimation method of robust regression developed by (Yohai, 1987). The MM-estimation method can simultaneously attain high breakdown points and efficiencies (Alanamu and Oyeyemi, 2018). Yohai's MM estimator was the first estimate with a high breakdown point and high efficiency under normal error (Alma, 2011). The efficiency is used to measure the relative accuracy of the robust estimate compared to the OLS estimate when the error distribution is exactly normal and there are no outliers. The breakdown point is used to measure the proportion of outliers an estimate can tolerate before it goes to infinity. The MM-estimation procedure is to estimate the regression parameters using S-estimation which minimizes the scale of the residual from M-estimation and then proceeds with M-estimation. The M-estimation method,

introduced by (Huber, 1964) is an extension of the maximum likelihood estimate method which minimizes some function of the residuals. The M in M-estimation stands for "maximum likelihood type". While the regression estimators associated with M-scales are the S-estimators proposed by (Yohai, 1987), the S-estimation method is based on a residual scale of the M-estimation method. The ''MM'' in MM-estimation referred to as more than one M-estimation procedure is used to calculate the final estimates. The MM-estimators have become increasingly popular and perhaps now the most commonly employed robust regression technique. Almetwally and Almongy (2018), discussed the comparison between the robust methods, where they observed that the S estimation, M-estimation, MM(S) and MM- estimation methods are the most efficient compared with other robust methods.

Inflation

Inflation can be defined as the rise in the level of prices maintained over a given period in an economy. In other words, it refers to the general rise in the price of various goods and services thus leading to a fall in the purchasing power of a country's currency (Lipsey and Crystal, 1995). In Nigeria, inflation has become a threat to the economy, particularly to workers whose standard of living gradually fell. The Inflationary trend in Nigeria is closely linked to the rise in the oil price over the years. It started in the early 1980s when prices of petrol shoot up from 9.5k to 15.4k per litre. This increase in the price of petrol led to an increase in the price of transport fares, foodstuffs, building materials, rents, goods and services. The inflation was further aggravated by the increase in the price of petrol from N97.00 per litre in 2012 and N145 per litre in the mid of 2016. Recently in the first quarter of 2020, the federal government reduced the price of petrol to N125 per litre due to Corona virus pandemic which affects the global economy that led to a crash in crude oil prices at the international market later in the second quarter of 2020, the price was increased again to N143 per litre and also in the last quarter of 2020, the price of petrol has increased from N143 to N145, N160, N172 and now N190 by the Federal Government of Nigeria. To promote economic growth and strengthen the purchasing power of the domestic currency for the Nigerian economy, emphasis has been laid by the Central Bank of Nigeria on maintaining stability in prices through the use of expansionary or contractionary monetary policy (Umaru and Zubairu, 2012).

Khan et al. (2021) claim that "a literature search shows that robust regression techniques are rarely used in applied econometrics, despite the weaknesses of the traditionally used OLS method. Based on our literature review, we found that many researchers have worked on the Inflation-Growth nexus in Nigeria and internationally, but none of them has used the robust method of regression in estimating the parameters of Inflation-Growth nexus models until 2019. Idris and Suleiman (2019), Improved the performance of ordinary least squares(OLS) using the MM-estimation method of robust regression on the analysis of the effects of inflation on the economic growth of Nigeria from 1980-2017 due to the inefficiency of the OLS in the presence of outliers. The MM-estimation method has an advantage over the OLS because it minimizes the function of the residuals using a weighting function that tackled the effects of outliers in the estimation process, unlike in the OLS that a single outlier is capable of deviating a large number of observations due to it low break down point. But on the other hand, the M-estimation method is only resistant to outliers in the response variable, so when there is a presence of outliers on the explanatory variables (leverage points) it is inefficient and it will also give a misleading result as the OLS. Hence, this study intends to improve the situation using the MM-estimation method of robust regression, which is resistant to outliers

both in the response and explanatory variables for the analysis of the inflation-growth nexus in Nigeria.

The study covered a period of forty (40) years from 1980 – 2019. The variables on which data were collected include Gross Domestic Product (GDP) which has been used as a proxy for economic growth and the dependent variable, while the Inflation rate (INFR), an Exchange rate (EXC) and Interest rate (INT) were used as the independent variables. The period was chosen because it was informed by the fact that the problem of inflation becomes more pronounced in the early 1980s following the increase of oil prices in the international market. Concerning, the significance of the study, the MM-Estimation robust regression method that was used in this study has the advantage of coping with outliers both in the dependent and independent variables unlike the M-estimation method that can only cope with vertical outlies and also over the OLS technique which cannot cope with any type of outlier contamination. Furthermore, the findings of the study will help policymakers in formulating policies that will help combat inflation and ensure sustainable economic growth in Nigeria.

Empirical Review

Here it is reviewed empirical works related to inflation and economic growth locally and internationally. Chude and Chude (2015) employ time-series data from 2000 to 2009 using ordinary least squares regression estimation techniques to examine the influence of inflation on the economic growth of Nigeria. The result indicates the positive and significant relationship between inflation, exchange rate and growth of the economy. Olu and Idi (2015), using the least squares method, analyzed the influence of inflation on the economic growth of Nigeria from 1980 to 2013. The result shows an insignificant positive relationship between the two variables. Enejoh and Tsauni (2017) examined how the inflation rate affects the country's economy using ARDL techniques and Granger causality during 47 years (1970–2016). The result indicates that the inflation rate and exchange rate have a positive impact on economic growth, while the lagged value of the exchange rate indicates a negative relationship with the growth of the economy.

Anidiobu et al., (2018) determined the influence of inflation on the economic growth of Nigeria using descriptive and ordinary least squares on the data for the period 1986–2015. The result indicates that the inflation rate depicts an insignificant positive relationship, the exchange rate shows a significant positive relationship, while there is a negative insignificant relationship between the interest rate and the growth of Nigeria's economy. Mohammed et al (2019) analyze the effect of unemployment and inflation on Nigeria's economic growth (GDP) from 1985 – 2017. Secondary Data was used, and the relevance of the Ordinary Least Square (OLS) technique was also used in determining the effects of unemployment and inflation on GDP. The results of OLS from the model indicate that unemployment and inflation have an insignificant relationship with GDP which implies that with the increase in GDP, ceteris paribus inflation rate will increase, and so also the unemployment rate. Idris and Suleiman (2019) examine the effect of inflation on the economic growth in Nigeria over thirty-eight (38) years from 1980 – 2017 using annual time series data on the real interest rate (RINR), the exchange rate (EXCR), inflation rate (INFL) and gross domestic product (GDP). The study employed the MM-estimation method of robust regression. Findings reveal a long-run relationship among the variables and the outcome of the robust regression estimate indicates that inflation and interest rate were negatively signed. That is the inflation rate and interest rate affect the economic growth of Nigeria significantly.

Dinh (2020) investigates the relationship between the inflation rate and economic growth from 1996 to 2017 to find out the optimal inflation threshold for economic growth. The study applied an ordinary least squares model (OLS). The study showed that 96.6% of the correlation between the inflation rate and economic growth is close and 4.5% of the optimal inflation threshold is appropriate for economic growth. It finds that the optimal inflation threshold is based to perform economic growth, and the inflation rate is positively related to economic growth. Begashaw and Yohannes (2020) applied the M-estimation method of robust regression in identifying outlies on the national growth of 61 countries from 1960-1985 using GDP as the dependent variable while labour force, equipped investment and noninvestment as independent variables. Their result reveals that Robust regression methods have much to offer a data analyst and its extremely helpful in locating outliers and highly influential observations. They also reveal that whenever a least squares analysis is perfumed it would be useful to perform a robust fit also. If the results of both the fit are in substantial agreement, the use of the Least square procedure offers a good estimation of the parameters. Khan et al (2021), improved the performance of the OLS technique using the Least trimmed square method of robust regression on the analysis of the impact of population growth on the economic growth of Pakistan and India for 41 years. Their results reveal that both techniques show a rapid population growth negatively influences the economic growth of both countries. They also explained the application of the LTS technique reveals that the overall fit of the model improves.

Materials and Methodology

Sources and Methods of data collection

The study used annual time series secondary data covering a period of forty years from 1980 to 2019. The data were collected from the publications of the Nigerian National Bureau of Statistics (NBS) and Central Bank of Nigeria (CBN) statistical bulletins of various years.

Model Specification

In this study, the RLS regression model was employed to show the relationship between inflation and economic growth in Nigeria. The model is stated as follows:

$$logGDP = \alpha + \beta log\ INF + \gamma\ logEXC + \lambda log\ INT + \mu_i \quad \ldots (1)$$

Where:
GDP = Real Gross Domestic Product
INT =Real Interest Rate
EXC =Real Exchange Rate
INF = inflation rate
α = intercept parameter
$\beta, \gamma\ and\ \lambda$ = partial slope parameters.
u_i = error term

Methods of model estimation

Ordinary Least squares regression method(OLS)

The general linear regression model is given by:

$$y = x\beta + e \quad \ldots (2)$$

Where Y is an $(n \times 1)$ response vector, X is an $(n \times p)$ matrix of n observations of p predictors. It is important to note that X is not a square matrix since the number of data values 'n' is usually larger than the number of predictors 'p'. β is a $(p \times 1)$ vector of unknown regression parameters, and e is an $(n \times 1)$ vector of the random noise in the observed data vector Y, it is often assumed that they are distributed as Gaussian with:

$$E(e) = 0 \quad \ldots (3)$$

And

$$v(e) = \delta^2 \quad \ldots (4)$$

However, a method is needed to estimate the parameter vector β. The most common method

is the least squared regression by finding the parameter values which minimize the sum of squared residuals given by:

$$SSR = \sum_{i=0}^{n}(y - x\beta)^2 \quad \ldots (5)$$

The solution turns out to be a matrix equation which is the OLS, defined by:

$$\hat{\beta} = (X'X)^{-1}X'Y \quad \ldots (6)$$

Where X' is the transpose of the matrix **X** and the exponent (−1) indicates the inverse matrix of the given quantity.

MM-Estimation

The MM-estimator is the solution of

$$\sum_{i=1}^{n}\rho_1(u_i)X_{ij} = 0 \; or \; \sum_{i=1}^{n}\rho_1\left(\frac{y_i - \sum_{j=0}^{k} X_{ij}\widehat{\beta}_j}{S_{MM}}\right)X_{ij} = 0 \quad \ldots (7)$$

where S_{MM} is the standard deviation obtained from the residual of S estimation and ρ is a Tukey's biweight function given as $\rho(u_i) =$

$$\begin{cases} \frac{\mu_i^2}{2} + \frac{\mu_i^4}{2C^2} + \frac{\mu_i^6}{6C^2}, & |u_i| \le C; \\ \frac{C^2}{6}, & |\mu_i| > C \end{cases}$$

Where C=4.685

Unit Root Test

To make inferences on time series data, they must be stationary. For this study, Augmented Dickey-fuller(ADF) unit root test will be used to test for the stationarity of the variables. Primarily under the test for unit root, two hypotheses are established in the ADF. The null hypothesis states that there is a unit root, meaning the series is not stationary, meanwhile the alternative states that there is no unit root meaning the series is stationary. Contrarily, in a case where the null hypothesis is rejected at level order (i.e. *=0), the next would be to take the first difference of the series to give us a stationary process in the series. In the case where the null is rejected, it means that the alternative hypothesis is accepted, it means the series is stationary at the first difference I (1).

Results and Discussions

Unit Root Test

The datasets for all the variables under study were tested for the existence of unit root using the Augmented Dickey-Fuller unit root test and the results are presented below:

Table 1 Augmented Dickey-Fuller (ADF) Unit Root Test Results

Variables	ADF	P value
GDP	-6.7046	0.035
INF	-4.4413	0.01
EXC	-4.9035	0.01
INT	-3.7358	0.01

Source: Authors' computation aided by R package v 4.1.1. GDP represents Real Gross domestic product, INF represents Inflation rate, EXC represents Exchange rate and INT represents Interest rate respectively.

Table 2 OLS Estimation Result

Variables	Coefficient	S.E	t-value	p-value
Constant	7.20376	0.07943	90.696	< 2e-16 ***
INF	-0.06081	0.06521	-0.933	0.357
EXC	0.21142	0.02275	1.704	4.22e-11 ***
INT	0.04451	0.02612	9.295	0.097
R^2	0.77			
Adj. R^2	0.75			
RSE	0.1205			

Source: Authors' computation aided by R package v 4.1.1. Note: (***) denote significance at 1

Outlier Detection and checking of some classical assumptions

Outliers and violations of distributional assumptions such as normality, linearity, and homoscedasticity are common in many areas of research. These issues might introduce substantial bias in the analysis and potentially lead to grossly incorrect inferences. The ordinary least squares method is quite sensitive for outlying observations, both for dependent and independent variables and can hurt the estimate. In higher dimensional data, these outlying observations can remain unnoticed. So, after running a regression analysis, you should check if the model works well for the data. One of the ways to check if the model works well is by plotting the residuals, it shows how poorly a model represents data and could also reveal unexplained patterns in the data by the fitted model. Therefore, we are going to plot the residuals of the regression estimate from Table 1 above to check for outliers, leverage points, influential observation and classical assumptions.

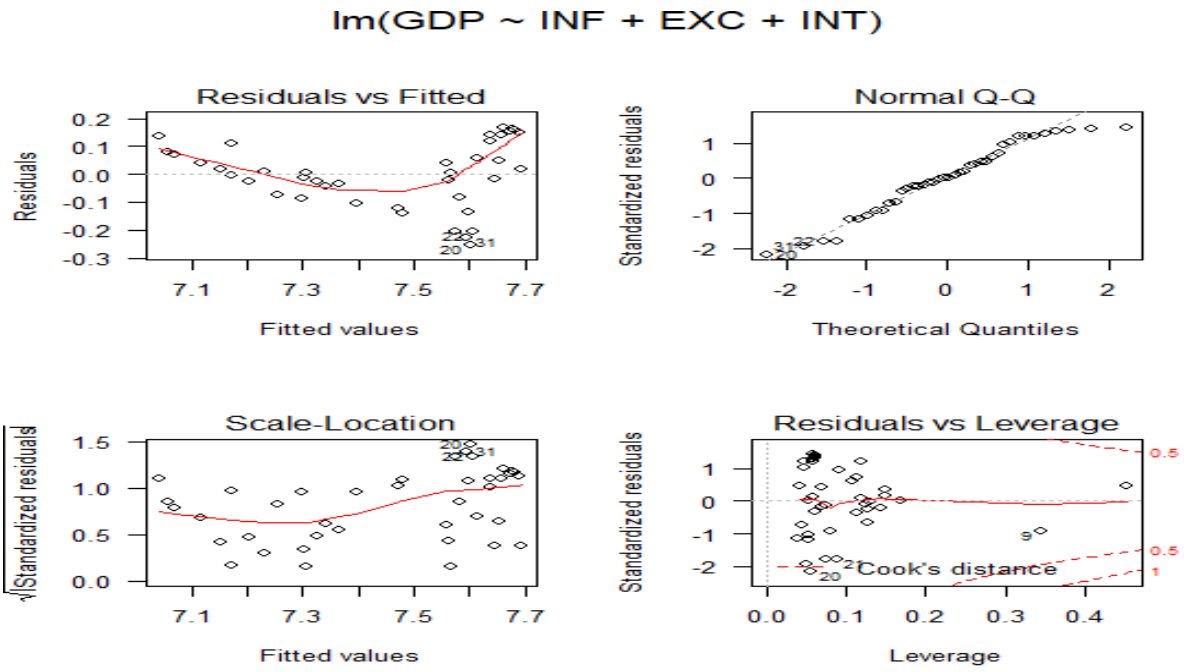

Figure 1 Residual Plots

Source: Authors' computation aided by R package v 4.1.1.

Based on the residuals diagnostic we made above, we conclude that there were violations of some classical assumptions, There was also the presence of leverage points and influential observation in the economic data which may affect the regression coefficients. When data is contaminated with outliers or there is influential observation in the data, the OLS method is inefficient and this reason is what led to the development of robust regression. Robust regression is an alternative to least squares regression when data are contaminated with outliers, influential observations or there are violations of some of the classical assumptions.

Table 3 MM-Estimation Result

Variables	Coefficient	S.E	t-value	p-value
Constant	7.22012	0.04852	148.803	< 2e-16 ***
INFR	-0.07551	0.04415	-1.710	0.0958.
EXCR	0.21638	0.01910	11.331	1.99e-13 ***
INTR	0.04205	0.02010	2.092	0.0435 *
$\mathbf{R^2}$	0.78			
$\mathbf{Adj.\,R^2}$	0.76			
RSE	0.1063			

Source: Authors' computation aided by R package v 4.1.1. Note: (*) and (***) denote significance at 10% and 1% respectively.

Discussion of Results

Table 1 displays the unit root test result and reveals that all the variables were stationary at level form i.e. I (0) and there is no further need to proceed to the first difference. Model estimation relating to time series data that are not stationary is sure to produce spurious results. As can be seen from the table above, the calculated values are more negative than the critical values for each of the variables tested. Therefore, all the variables attained stationarity at 5% critical values. From the result of the OLS in table 2, we can see that Inflation was found to have a negative but insignificant effect on the economic growth of Nigeria while exchange rate and interest were found to be positively related to the economic growth. Looking at the result of the regression analysis using the MM-Estimation Method presented in table 3, Inflation was also found to be negatively related to the economic growth of Nigeria as the OLS but here it was found to be significant at 10%. We can also see that the Residual Standard Error(RSE) of the MM-estimation (0.1063) is smaller than the RSE (0.1205) of the OLS and also the adjusted $R^2$of the MM-Estimation (0.76) is greater than the adjusted R^2 of the OLS (0.75) meaning that the MM-Estimation model fit the data well more than the OLS, and this is because of the high breakdown point and bounded influence of the MM-Estimation technique that can cope with the presence of outliers unlike the OLS that a single outlier is capable of deviating a large number of observations due to it low break down point. The MM-Estimation result shows that a unit increase in INF decreases GDP by about 7.55 per cent. This conforms to the prior expectation and supports the classical theory of Economics that inflation is a disaster to any economy where it exists and it reduces the purchasing power of economic agents and aggregate demand thereby hurting economic growth which is in line with the findings of Mamo (2012), Nazir et al (2017), Idris and Suleiman (2019), Adaramola and Dada (2020) and Akintola and Cole (2022). While exchange rate and interest were found to have a positive and significant impact on the economic growth of Nigeria meaning that a unit increase in the

EXC will cause an increase of 21.64 percent in GDP and a unit increase in INT will cause an increase of 4.20 percent in GDP respectively. The application of MM-estimation reveals that the overall fit of the model improves, and the significance of the predictors changes significantly compared to the OLS model.

Conclusion

Based on the findings of this study, we can see that the OLS is highly sensitive to the presence of outliers be it vertical or horizontal, meaning that, using the OLS in data that contains outliers or influential observations can give a misleading result. Therefore, from the result of this study, we conclude that the appropriate method of regression estimation to be used when there is outlier contamination, particularly for the analysis of the effects of inflation on the economic growth of Nigeria, is the MM-estimation method because of its high break down point and high efficiency which can cope with the presence of outliers in the economic data and will also correct the violation of the classical assumptions if any.

Recommendations

To tackle the problem of outlier contamination affecting econometric models, particularly in determining the nature of the relationship that exists between inflation and economic growth of Nigeria and other inflation-growth nexus analyses, which led to so many controversies results among Econometricians regarding the nature of the relationship that exists between inflation and economic growth of Nigeria. we recommend that econometricians should adopt the use of the MM-estimation method of robust regression as an alternative to the traditionally used OLS technique.

References

Abonazel, M., & Rabie, A. (2019). Impact of using robust estimation in regression models: An application of Egyptian economy. *Journal of Advanced Research in Applied Mathematics and Statistics,*4, 8-16.

Adaramola, O.A., & Dada, O. (2020). Impact of inflation on economic growth: Evidence from Nigeria. *A journal of Investment Management and Financial Innovations*, 17(2), 1-13.

Akintola, A.F., & Cole, A.A. (2022). Inflation and economic growth in Nigeria. *Global Scientific journal*, 1(1), 1039-1046.

Almetwally, E. M., & Almongy, H. M. (2018). Comparison between M-estimation, S-estimation, and MM-estimation methods of robust estimation with application and simulation. *International Journal of Mathematical Archive,* 9(11), 55-63.

Alma, Ö. (2011). Comparison of robust regression methods in linear regression. *International Journal of Contemporary Mathematical Sciences*, 6(9), 409-21.

Alanamu, T., & Oyeyemi, G.M. (2018). A new robust method for estimating linear regression model in the presence of outliers. *Pacific Journal of Science and Technology,* 19(1), 125-132.

Anidiobu, G. A., Okolie, P. I. P., & Oleka, D. C. (2018). Analysis of inflation and its effect on economic growth in Nigeria. *Journal of Economics and Finance, 9*(1), 28-36.

Begashaw, A.B., & Yohannes, Y.B. (2020). Review of outlier detection and identification using robust regression. *International Journal of Systems Science and applied mathematics,* 5(1), 4-11.

Chude, D. I., & Chude, N. P. (2015). Impact of inflation on economic growth in Nigeria.

International Journal of Business and Management Review, 3(5), 26-34.

Dinh, D.V. (2020). Optimal inflation threshold and economic growth: Regression model analysis. *Journal of Asian Finance, Economics and Business.* 7(5), 091 – 102.

Enejoh, S. Y., & Tsauni, A. M. (2017). An analytical study of the impact of inflation on economic growth in Nigeria (1970–2016). *International Journal of Academic Research in Accounting, Finance and Management Sciences, 7*(4), 110-120.

Huber P.J. (1964). Robust version of a location parameter. *Annals of Mathematical Statistics*, 36, 1753–1758.

Idris, T. S., & Suleiman, S. (2019). Effect of inflation on economic growth in Nigeria: 1980-2017. *MAJASS,* 18, 33-48.

Khan *et al*. (2021). Applications of robust regression techniques: An econometric approach. *Journal of Hindawi Mathematical Problems in Engineering*, 9(1), 1-9.

Lipsey R.G., & Chrystal K.A. (1995). *An introduction to positive economics.* 8th edition. New York: Oxford: Oxford University Press.

Mamo, F. (2012). Economic growth and inflation*:* A panel data analysis. Södertörn University, Department of Social Sciences, Economics. Master's Programme Thesis.

Muhammed, AS., Usman, Y.A., & Sa'ad, S. (2019). Effects of unemployment and inflation on economic growth in Nigeria. *Dutse journal of economics and development studies(DUJED),*7(1), 113-123.

Nazir, S., Saira, S., & Atta, M. (2017). Threshold modelling for inflation and GDP growth. *A journal of MPRA Page 79649.*

Olu, J. F., & Idih, E. O. (2015). Inflation and economic growth in Nigeria. *Journal of Economics and International Business Management, 3*(1), 20-30.

Umaru, A., & Zubairu, A. A. (2012). Effect of inflation on the growth and development of the Nigerian economy (an empirical analysis). *International Journal of Business and Social Science, 3*(10), 183-191.

Yohai, V.J. (1987). High breakdown point and high efficiency robust estimates for regression. *Annals of Statistics*, 15(20), 642–656.